# Mat 2377: Quiz #4

July 27, 2016

Answer all the questions first in one 45 minute sitting. Then check your answers against the solutions given below.

1. Suppose we have a random sample $X_1, ..., X_{20}$ of size 20 from the uniform distribution on the interval (0,1). Let $Y = \sum_{i=1}^{20} X_i$. Calculate approximately

a) $P(Y \leq 9.1)$and

b) $P(8.5 \leq Y \leq 11.7)$

2. Suppose that we have a random sample of 25 from a distribution whose density is given by

$$f(x) = \frac{x^3}{4}, 0 < x < 2$$

Let $\bar{X}$ denote the sample mean. Compute approximately

$$P\left(1.5 \leq \bar{X} \leq 1.65\right)$$

3. Let $Y$ be a random variable having a binomial distribution with parameters $n = 36, p = \frac{1}{2}$.Compute approximately

$$P(12 < Y \leq 18)$$

4. Let $T$ have a Student t distribution with degrees of freedom equal to 7.

Calculate

$$P(-1.895 < T < 1.415)$$

5. Suppose that we wish to compare the grades for two independent sections of the same course follow normal distributions with the following information

| Section | $\mu$ | $\sigma^2$ | sample size |
|---|---|---|---|
| A | 81 | 60 | 30 |
| B | 78 | 50 | 25 |

Calculate

$$P(\bar{X} > \bar{Y})$$

6. Suppose that we have the same problem as in 5 except that the data is now

| Section | $\mu$ | $s^2$ | sample size |
|---|---|---|---|
| A | 81 | 60 | 30 |
| B | 78 | 50 | 25 |

Assuming that the variances $\sigma_1^2 = \sigma_2^2$, calculate

$$P(\bar{X} > \bar{Y})$$

7. The time required for maturation of a certain species of seeds in normally distributed with mean and variance respectively $\mu, \sigma^2 = 9$. A sample of 13 seeds is selected at random and yielded a mean and variance equal to 18.97 days and10.7 respectively. Is the observed value of variance reasonable?

Solutions

1. We first note that the mean and variance for a uniformly distributed variable is $\frac{1}{2}$ and $\frac{1}{12}$ respectively. Hence,

$$E[Y] = 20 \times \frac{1}{2}, Var[Y] = 20 \times \frac{1}{12}$$

a)

$P(Y \leq 9.1) = P\left(\frac{Y - \frac{20}{2}}{\sqrt{\frac{20}{12}}} \leq \frac{9.1 - \frac{20}{2}}{\sqrt{\frac{20}{12}}}\right) \approx P(Z \leq -0.697) = 0.2423$

b) In the same way as in a)

$P(8.5 \leq Y \leq 11.7) = P\left(-1.162 \leq \frac{Y - \frac{20}{2}}{\sqrt{\frac{20}{12}}} \leq 1.317\right) \approx P(-1.162 \leq Z \leq 1.317) = 0.9061 - 0.1226 = 0.7835$

2. We first compute the mean and variance from the density. It is seen that $\mu = \frac{8}{5}, \sigma^2 = \frac{8}{75}$. Hence,

$$E\left[\bar{X}\right] = \frac{8}{5}, Var\left[\bar{X}\right] = \frac{1}{25} \times \frac{8}{75}$$

$$\begin{aligned} P\left(1.5 \leq \bar{X} \leq 1.65\right) &= P\left(\frac{1.5 - \frac{8}{5}}{\sqrt{\frac{1}{25} \times \frac{8}{75}}} \leq \frac{\bar{X} - \frac{8}{5}}{\sqrt{\frac{1}{25} \times \frac{8}{75}}} \leq \frac{1.65 - \frac{8}{5}}{\sqrt{\frac{1}{25} \times \frac{8}{75}}}\right) \\ &\approx \Phi(0.765) - \Phi(-1.531) \\ &= 0.7779 - 0.0629 = 0.7150 \end{aligned}$$

3. We use the normal approximation to the binomial which is a discrete distribution. We note that $E[Y] = np = 18, Var[Y] = np(1-p) = 9$.

$P(12 < Y \leq 18) = P(12.5 \leq Y \leq 18.5) = P\left(\frac{12.5-18}{\sqrt{9}} \leq \frac{Y-18}{\sqrt{9}} \leq \frac{18.5-18}{\sqrt{9}}\right) \approx \Phi(0.167) - \Phi(-1.833) = 0.5329$

4. From Table A.4 P.439, we first note that the T distribution is symmetric

around 0 just like the normal. Hence,

$$F(t) = 1 - F(-t)$$

Also

$$F(1.415) = P(T \leq 1.415) = 0.90$$

$$\begin{aligned} P(-1.895 < T < 1.415) &= F(1.415) - F(-1.895) \\ &= F(1.415) - [1 - F(1.895)] \\ &= 0.90 - 0.05 = 0.85 \end{aligned}$$

5. We first note that the distribution of $\bar{X} - \bar{Y}$ is Normal with mean $=$ $(\mu_X - \mu_Y)$ and variance $= \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$

$$\begin{aligned} P\left(\bar{X} > \bar{Y}\right) &= P\left(\bar{X} - \bar{Y} > 0\right) \\ &= P\left(\frac{\bar{X} - \bar{Y} - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} > \frac{-(\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}\right) \\ &= P\left(Z > \frac{-3}{\sqrt{\frac{60}{30} + \frac{50}{25}}}\right) \\ &= P(Z > -1.5) = 0.9332 \end{aligned}$$

6. Since the estimates of the variances are used we note that the distribution of

$$T = \frac{\bar{X} - \bar{Y} - (\mu_X - \mu_Y)}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

is that of a student t with with $n_1 + n_2 - 2$ degrees of freedom where the pooled estimate of common variance is equal to

$$s^2 = \frac{(n_1 - 1)\, s_1^2 + (n_2 - 1)\, s_2^2}{n_1 + n_2 - 2}$$

In our case

$$s^2 = \frac{29\,(60) + 24\,(50)}{53} = \frac{2940}{53}$$

Hence

$$\begin{aligned} P\left(\bar{X} > \bar{Y}\right) &= P\left(\bar{X} - \bar{Y} > 0\right) \\ &= P\left(T > \frac{-\left(\mu_X - \mu_Y\right)}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}\right) \\ &= P\left(T > \frac{-3}{2.0115}\right) \\ &= P\left(T > -1.49\right) \end{aligned}$$

This probability is between 0.90 and 0.95

7. We know the statistic $\frac{(n-1)s^2}{\sigma^2}$ folows a Chi square with $n-1$degrees of freedom. So we calculate the value of that statistic

$$\frac{12\,(10.7)}{9} = 14.27$$

and look to see what is the probability of observing a value at least as large using the Chi table A.5 P.442. We see the probability is between 0.25 and 0.30. We conclude that the value is resonable